

ON THE VERIFICATION OF INTER-UNIVERSAL  
TEICHMÜLLER THEORY: A PROGRESS  
REPORT (AS OF DECEMBER 2013)

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Nearly a year and four months have elapsed since I released my series of papers on inter-universal Teichmüller theory (IUTEich) in August 2012. In the following, I would like to report on various developments that have come to pass during this time with regard to the verification of IUTEich.

(1) At the end of August 2012, I released my series of four papers concerning IUTEich and submitted them to a mathematical journal. I have no intention of disclosing the name of the journal or other information related to this journal submission. The papers were released in order to make it possible for *specialists* to *study* and *assess the validity* of IUTEich; they were *never intended* as an *announcement to the general public*. Moreover, I never envisaged that these papers might elicit *reactions* consisting of *non-mathematical content* from *non-specialists*. I have *no intention* of *responding* to such non-mathematical reactions.

(2) As of the time of writing, I have given *two talks* on IUTEich at *Kyoto University* — in

October 2010 (i.e., a one-hour “preview” talk) and  
December 2012 (one hour)

— and *one talk* on IUTEich at the *University of Tokyo* — in

June 2013 (one and a half hours).

The slides for these talks are available on my web site (on the page entitled “Travel and Lectures”). It is likely that I will again give at least one talk at some academic institution in Japan in 2014. The various talks that I have given so far cover essentially the same content; this content is exposed in more detail in the survey [Pano]. This survey is available on the page of my web site entitled “Papers” and is scheduled to appear in the proceedings for the workshop at which my December 2012 talk was delivered.

(3) I have been conducting a *seminar* devoted to the *detailed examination* of IUTeich with *Go Yamashita* (Research Fellow at Toyota Central Research and Development Laboratories; Project Lecturer at the Center for Research Interaction in Mathematical Sciences of RIMS, Kyoto University)

once a month (for 2 days  $\approx$  12 hours), since October 2012.

More precisely, during the period

October  $\sim$  December of 2012,

Yamashita studied the “preparatory papers” (i.e., [HASurI], [HASurII], [Semi-Anbd], [FrdI], [FrdII], [EtTh], [AbsTopIII], [GenEll], etc.) and discussed their contents with me in our seminar. During the periods

January  $\sim$  March 2013 and

April  $\sim$  September 2013,

he then proceeded to *read through* the entire “*theory proper*” constituted by the four papers [IUTchI], [IUTchII], [IUTchIII], [IUTchIV] a total of *two times*. During this period of roughly a year, I received from Yamashita, in writing (by e-mail), numerous *detailed technical comments* (roughly several hundred during this period of a little over a year!) that exceed, by a substantial margin, the scope of a typical referee’s report for a mathematical journal. After discussing these comments in our seminar, I proceeded to correct the relevant papers and post the corrected versions on the page of my web site entitled “Papers”. Also, around April 2013, Yamashita prepared an “*FAQ*”, which I posted on my web site, to answer questions posed by other researchers with whom he had occasion to interact at workshops and so on. As a result of these two readings of the four papers, Yamashita acquired a thorough understanding of the theory and, in particular, verified the correctness of the theory, although, in his words, this verification constituted, for him, only a “*single check*” of the theory.

(4) Yamashita then proceeded to carry out, in his words, a “*double check*” of IUTeich. This “double check” consisted of a seminar that was conducted

once a month (for 2  $\sim$  3 days  $\approx$  16  $\sim$  24 hours), May  $\sim$  November 2013,

for the following three participants:

*Akio Tamagawa* (RIMS, Kyoto University, Professor)

*Yuichiro Hoshi* (RIMS, Kyoto University, Lecturer)

*Makoto Matsumoto* (Hiroshima University, Professor).

The seminar was conducted without my participation, in order to provide Yama-

shita with an opportunity to explain the theory in his own words and thereby verify his understanding of the theory. Yamashita’s presentation of the theory in this seminar consisted of literally wading through — from the most elementary portion of the “preparatory papers” to the final portion of the “theory proper” — each Definition, Proposition, Theorem, etc. in the order written in these papers — an approach that required quite a substantial amount of effort on the part of both the lecturer and the participants. When, either during the seminar or in the course of preparing for it, technical comments or questions arose (roughly 10 ~ 30 a month, on average), Yamashita and I discussed and resolved them the following month in our monthly seminar. Toward the end of the (“double check”) seminar, one participant made a remark to the effect that, although it seemed likely that it would take some time for him to digest the theory, he felt that he had more or less reached an understanding of the general outline of the theory. Moreover, the participants agreed that, from their present point of view, i.e., after having seen Yamashita’s presentation of the theory, the content of the lectures and survey discussed in (2) now seemed to be quite “appropriate”.

(5) Subsequent to the successful conclusion of the “double check” of (4), Yamashita has started writing a detailed *survey* of IU $\mathit{Teich}$ , i.e., which, in his words, will constitute a “*triple check*” of the theory. He expects the survey to be about 200 ~ 300 pages in length. Moreover, although his plans are not yet definite, he is considering, at the request of Associate Professor Yuichiro Taguchi of Kyushu University, giving a (several week long) intensive course on IU $\mathit{Teich}$  at Kyushu University, at some date subsequent to April 2014.

(6) *Mohamed Saïdi* (University of Exeter (UK), Associate Professor) visited RIMS, Kyoto University, as a Visiting Professor, for

the three month period of July ~ September 2013

during which I conducted a *seminar* with him devoted to the *detailed examination* of IU $\mathit{Teich}$  from various points of view. This seminar consisted of

roughly 10 meetings ( $\approx$  24 hours in all).

He also met with Yamashita a few times to discuss IU $\mathit{Teich}$ . During a period of a little over half a year prior to his visit, Saïdi apparently finished reading the “preparatory papers”, as well as roughly half of the “theory proper”. Upon arriving in Japan, he then finished reading the “theory proper”. During his visit, in order to verify his understanding of the theory, he then proceeded to read through the entire four papers of the “theory proper” once again. These two readings of the “theory proper” were supplemented by the seminars that I conducted with

him, roughly once a week during his visit, in which we discussed the content of the papers in quite substantial detail. During these seminars, he provided me with numerous *detailed technical comments* (roughly a hundred during the three months of his visit!) that exceed, by a substantial margin, the scope of a typical referee’s report for a mathematical journal. Just as in the case of Yamashita, after discussing these comments in our seminar, I proceeded to correct the relevant papers and post the corrected versions on the page of my web site entitled “Papers”. In conclusion, Saïdi stated to me, as well as, apparently, to at least one “third-party” researcher outside Japan, that he believed the theory to be correct.

(7) Although both Yamashita and Saïdi are researchers in the field of arithmetic geometry, they nonetheless come from substantially different mathematical backgrounds, as may be verified by a brief look at their previous papers and other work. One important attribute that they share, however, is their *proven track record as referees*, i.e., their wealth of experience in refereeing for mathematical journals and, in particular, being in a position to judge the suitability of submitted manuscripts for publication in such journals. On the other hand, in the case of IUTeich, in light of both the *importance of the theory* and the *novelty of the ideas* that constitute the theory, it is desirable that the verification process be conducted with the *utmost caution*. In particular, the issue of whether or not one should regard the verification of IUTeich as being, for all practical purposes, complete, i.e., as a result of the activities of Yamashita and Saïdi, is by no means clear, and any sort of “final conclusion” on this topic must be regarded as a matter that lies beyond the scope of the present report. Nevertheless, such caveats notwithstanding, it seems to me that the degree of meticulousness and attention to detail exhibited in the verification activities summarized above in (3), (4), (6) — which, as noted above, exceed, by a substantial margin, the scope of a typical referee’s report for a mathematical journal — together with the wealth of refereeing experience of Yamashita and Saïdi (i.e., as discussed above) should be regarded as lending *quite substantial weight* to the *extremely positive evaluation* that I received from both of them in the course of these activities.

(8) One further important “by-product” of the verification activities of Yamashita and Saïdi concerning IUTeich is the fact that their experiences demonstrated, in effect, in a very explicit fashion, that, despite the quite substantial differences in their mathematical backgrounds,

it is quite *possible* to achieve a *reasonably rigorous understanding* of the theory within a period of *a little less than half a year*.

Since there apparently exist researchers whose attempts to study IUTeich have not

proceeded so smoothly, I asked Yamashita and Saïdi on numerous occasions if they had any “advice” for such researchers. Despite numerous discussions concerning this topic, the only “advice” that I was able to elicit from them consisted of remarks to the effect that

*if one proceeds to study carefully step by step, starting from the “preparatory papers”, there is no reason that one should encounter any insurmountable difficulties.*

On the other hand, the potential reader should keep the following point in mind:

- Although various *limited portions* of IUTeich exhibit similarities to existing mathematical theories such as complex or  $p$ -adic Teichmüller theory and the functional equation of the classical theta function, any attempt to study IUTeich under the expectation that the *essential thrust* of IUTeich will proceed via a similar pattern of argument to existing mathematical theories is likely to end in failure.

Both Yamashita and Saïdi voiced similar opinions concerning this point. Moreover, over the past few years, I have frequently emphasized the following two points:

- Although many individuals appear to be interested mainly in the numerical aspects of the inequality of the ABC Conjecture, in fact it is more natural to regard the *theory* that constitutes *IUTeich* as the main object of interest. That is to say, the *inequality* of the ABC Conjecture is best understood as simply being *one barometer* of the depth of the mathematical content of the theory.
- Typically, a one-hour lecture — or, indeed, even a week-long exposition of the theory — will be far from sufficient for most researchers to arrive at a sufficiently motivating appreciation of the essential framework that underlies the theory. In particular, based on my past experiences, I harbor substantial doubts that such short-term promotion activities can lead to meaningful results. On the other hand, although the precise amount of time necessary to arrive at a reasonable understanding of the theory will no doubt vary from individual to individual, at least in the case of researchers familiar with conventional arithmetic geometry, the amount of time necessary should, as remarked above, be a *matter of months* — i.e., roughly *half a year* or so — *not a matter of years!*

One aspect of my interaction with Saïdi that left a particularly strong impression on me was the fact that he voiced opinions similar in content to these last two points as “independent observations”, i.e., without even the slightest bit of prompting on my part!

(9) Finally, I would like to take this opportunity to express my deep gratitude to Yamashita and Saïdi, as well as the participants of (4), for their exceptional and zealous contributions to the goal of verifying IU $\text{Teich}$ .

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